

Gravitational time dilation in comparison, Andreas Gimsa, 13 February 2015

Evidence of gravitational time dilation in principle: Pound-Rebka experiment
(Measurement using the Mößbauer effect at the Jefferson Tower at Harvard University, 1960)

The calculation shown here is based on the formulas in the book and supplies more accurate results than the calculation with the Einstein field equations.

1. General data

c	2,99790000000000E+08 m/s	Speed of light
γ	6,67430000000000E-11 m ³ /(kg s ²)	Gravitational constant
m ₁	5,97700000000000E+24 kg	Mass of the Earth
E ₂	2,30716800000000E-15 J (14.4 keV)	Photon energy gamma (iron 57)
m ₂	2,5671126998946E-32 kg	Photon mass equiv. m ₂ = E ₂ /c ²
r ₁	6,37800000000000E+06 m	Earth's radius
r ₂	0,00000000000000E+00 m	Photon radius (point mass)
h	4,50000000000000E+01 m	Redshift for 45 m Tower height = 22.5 m
r=r ₁ +r ₂ +h	6,37804500000000E+06 m	Centre distance between objects

2. Calculation of the Schwarzschild metric from Einstein's field equations

g	9,8064952287224E+00 m/s ²	$g = \gamma m_1 / r^2$
Φ^*	4,4129228529251E+02 m ² /s ²	$\Phi = g h$ (grav. potential: Newton)
t/t ₀	1,00000000000000E+00	$t / t_0 = (1 + 2 \Phi / c^2)^{1/2}$
t _{2a} = t ₀	3,15360000000000E+07 s	for example, one year
τ	3,15360000000000E+07 s	
$\Delta t_2 = \tau - t_0$	1,5273690223694E-07 s	
$\Delta t_2/t_{2a}$	4,8432553981779E-15	corresponds to $\Delta f/f$

3. Gravitational time dilation from "The beauty of nature"

t _{2a}	3,15360000000000E+07 s	One year
Δt_2 S.13 [03-16]**	-1,5484661713977E-07 s	Time dilation during over this period
$\Delta t_2/t_{2a}$	-4,9101540188918E-15	corresponds to $\Delta f/f$

$$\Delta t_2 = - \frac{t_{2a}}{\frac{c^2 r(r-h)(m_{1a} + m_{2a})}{h m_{1a}^2 \gamma} + 1}$$

4. Measured results from the 1960 Pound-Rebka experiment (blue shift downwards)

$\Delta f/f$ (= $\Delta t_2/t_{2a}$)	-5,13000000000000E-15	Error: (+/-) 0.51 E-15
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* The calculation fails with great heights and assumes that the second mass is small compared to the first (lack of momentum conservation).

** Prerequisites for this formula: Energy conservation [03-02], momentum conservation [03-05], mass decay [02-01], origination of time [06-06], constant effect (entropy) [02-06], dilated information in accordance with [16-04]